Twistor–like superparticles revisited

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Abstract

We consider a formulation of N=1 D=3,4 and 6 superparticle mechanics, which is manifestly supersymmetric on the worldline and in the target superspace. For the construction of the action we use only geometrical objects that characterize the embedding of the worldline superspace into the target superspace, such as target superspace coordinates of the superparticle and twistor components. The action does not contain the Lagrange multipliers which may cause the problem of infinite reducible symmetries, and, in fact, is a worldline superfield generalization of the supertwistor description of superparticle dynamics.

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1 Introduction.

Twistor–like doubly supersymmetric formulations of superparticles [1]–[7], superstrings [8, 9, 11] and supermembranes [12] have attracted considerable attention, in particular, because of a hope to break through the long–standing problem of the covariant quantization of these theories.

In the twistor–like approach the infinite reducible fermionic κ–symmetry [13, 14], which causes the problem of covariant quantization [15], is replaced by local worldsheet supersymmetry which is irreducible by definition, and the theory is formulated as a super-field supergravity theory in a worldsheet superspace embedded into a space–time target superspace. Thus, the model of this kind possesses double supersymmetry.

Earlier doubly supersymmetric dynamical systems (of more general physical contents) were considered by several groups of authors with the aim to find a version of the super-string theory where the Neveu-Schwarz-Ramond and Green–Schwarz formulation would appear as a different choice of gauge [16, 2, 3, 8].

Several versions of twistor–like doubly supersymmetric particles and heterotic strings have been constructed in $D = 3, 4$ and 6 dimensions of space–time, while in D=10 only one superfield formulation is known [7] and, unfortunately, the latter itself suffers the infinite reducibility problem of a new local symmetry [7] being crucial for the possibility of eliminating auxiliary degrees of freedom. Note, however, that at the component level, when auxiliary fields were eliminated by gauge fixing and solving for relevant equations of motion, all remaining local symmetries are irreducible. This also takes place in a twistor–like Lorentz–harmonic formulation of super–p–branes [17, 18],which has been developed in parallel with the superfield twistor approach, and which is, in fact a component version of the latter [19, 20].

The existence of reducible symmetries in twistor–like formulations of super–p–branes considered so far is connected with the structure of the Lagrangian, which is constructed with the use of Lagrange multipliers, and, in general, it is difficult to endow the latter with a reasonable geometrical and physical sense. At the same time their role in the action is to provide geometrical conditions, which characterize the properties of the embedding of super–p–brane world surface into target superspace. The most essential trouble which happens with Lagrange multipliers is that part of them become propagative fields in the case of N=2, D=10 superstrings and N=1, D=11 supermembranes [11, 12], which spoils the physical contents of the theory. In this situation it seems reasonable to revise the way the superfield formulation of super–p–branes is constructed by use of well defined geometrical objects on world supersurface and target superspace, and the twistors (or harmonics) are among them.

So the motivation of the present paper is to develop a version of the twistor–like formulation which would be free of the reducibility problem already at the superfield level, and, would look "twistor–like" as much as possible. The letter, as we hope, may allow one to better utilize the powerful twistor techniques in the structure of supersymmetric theories.

As an example of such alternative formulation we consider the case of massless N=1

superparticles [30] in D=3,4 and 6 superspace–time, and as a basis for the doubly super-symmetric generalization we use a (super)twistor formulation of superparticle dynamics originated from papers by Ferber [23] and Shirafuji [24].

The superfield twistor–like models of $N = 1$ superparticles in $D = 3, 4, 6$ and 10 considered so far are based on the doubly supersymmetric generalization of the following massless bosonic particle action [1]:

$$S = \int d\tau p_m(\dot{x}^m - \bar{\lambda}\gamma^m\lambda), \tag{1}$$

where p_m is the particle momentum and λ^α is a commuting spinor variable ensuring the validity of the mass shell condition $p_m p^m = 0 = \dot{x}_m \dot{x}^m$ due to the Cartan–Penrose representation $\dot{x}^m = \bar{\lambda}\gamma^m\lambda$ of the light–like vectors in $D = 3, 4, 6$ and 10 space–time dimensions (m=0,...D-1; α=1,...,2(D-2)).

The straightforward doubly supersymmetric generalization of (1) is [1, 7]

$$S = \int d\tau d^{D-2}\eta P_{mq}(D_q X^m - iD_q\bar{\Theta}\gamma^m\Theta), \tag{2}$$

where the number $n = D - 2$ of the local worldline supersymmetries (q=1,...D-2) is equal to the number of the κ–symmetries in $D = 3, 4, 6$ and 10, and is half the number of Θ; $D_q = \frac{\partial}{\partial\eta^q} + i\eta_q\partial_\tau$ is an odd supercovariant derivative in a worldline superspace (τ, η^q) ($\{D_q, D_p\} = 2i\delta_{pq}\partial_\tau$), and (X^m, Θ^α) are worldline superfields which parametrize the "trajectory" of the superparticle in a target superspace. Bosonic spinor variables λ^α_q appear in (2) as superpartners of Grassmann coordinates $\theta^\alpha = \Theta^\alpha|_{\eta=0}$:

$$\lambda^\alpha_q = D_q\Theta^\alpha(\tau, \eta)|_{\eta=0} \tag{3}$$

The analysis of the action (2) [1, 7] shows that it describes a superparticle classically equivalent to the massless $N = 1$ Brink–Schwarz superparticle in $D = 3, 4, 6$ and 10 .

As we have already mentioned, in $D = 4, 6$ and 10 the action (2) possesses a local symmetry [7] under the following transformations of the Lagrange multiplier P_{mq} :

$$\delta P_{mq} = D_p\bar{\Xi}_{qpr}\gamma_m D_r\Theta, \tag{4}$$

with $\bar{\Xi}^\alpha_{qpr}$ being symmetric and traceless with respect to the indices (p, q, r). This symmetry is infinite reducible since P_{mq} is inert under the transformations (4) with

$$\bar{\Xi}^\alpha_{qpr} = D_s\bar{\Xi}^\alpha_{qprs} \tag{5}$$

where $\bar{\Xi}^\alpha_{qprs}$ is again symmetric and traceless, and (5) is trivial if $\bar{\Xi}^\alpha_{qprs} = D_s\bar{\Xi}^\alpha_{qprst}$ and so on and so far.

The reducibility of the transformations (4) is akin to the reducibility of the gauge symmetries of the antisymmetric gauge fields. It is just the problem of reducible symmetries in these theories that stimulated further development of the quantization procedure which was consistently followed for finite reducible symmetries [21] to which the gauge transformations of the antisymmetric bosonic tensor fields belong. However, the general

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receipt for dealing with the infinite reducible symmetries is still unknown (see [22] and refs. therein). Thus, one has to avoid this problem one way or another. In the case under consideration we may try to find another form of the twistor–like superfield action. To this end let us start with

2   D=3, N=1, n=1 superparticle.

The bosonic particle action is chosen to be [23, 24]

$$S = \int d\tau \bar{\lambda}\gamma_m \lambda \dot{x}^m, \tag{6}$$

To generalize (6) to the doubly supersymmetric case one could naively try (using (3)) to write down an action in the following form

$$S = \int d\tau d\eta D\Theta_\alpha D\Theta_\beta DX^{\alpha\beta}, \tag{7}$$

Where $X^{\alpha\beta} \equiv X^m \gamma_m{}^{\alpha\beta}$.

However, the action (7) does not describe ordinary $N = 1$, $D = 3$ superparticle. The reason is that (7) is invariant under transformations

$$\delta\Theta^\alpha = \epsilon_1^\alpha, \qquad \delta X^{\alpha\beta} = \Theta^\alpha \epsilon_2^\beta + \Theta^\beta \epsilon_2^\alpha,$$

so that the target space is not the usual superspace, but one with additional θ–translations.

Note that action (7) is part of a so called spinning superparticle model considered several years ago [16, 2, 4].

To construct a doubly supersymmetric action for an $N = 1$ superparticle we have to keep only one target space supersymmetry. The right action turns out to be as follows [31]

$$S = \int d\tau d\eta \Lambda_\alpha \Lambda_\beta (DX^{\alpha\beta} - iD\Theta^\alpha \Theta^\beta - iD\Theta^\beta \Theta^\alpha), \tag{8}$$

where $\Lambda_\alpha(\tau, \eta)$ is a commuting spinor superfield (compare with [23, 24]).

In addition to $N = 1$ target space supersymmetry and $n = 1$ local worldline supersymmetry

$$\delta\eta = \frac{i}{2}D\Xi(\tau, \eta), \qquad \delta\tau = \Xi + \frac{1}{2}\eta D\Xi, \qquad \delta D = -\frac{1}{2}\partial\Xi D \tag{9}$$

the action (8) is invariant under bosonic transformations

$$\delta X^{\alpha\beta} = b(\tau, \eta)\Lambda^\alpha \Lambda^\beta, \qquad \delta\Theta^\alpha = 0 = \delta\Lambda^\alpha \tag{10}$$

and under a superfield irreducible counterpart of the conventional fermionic κ–symmetry

$$\delta\Theta^\alpha = \kappa(\tau, \eta)\Lambda^\alpha, \qquad \delta X^{\alpha\beta} = 2i\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \qquad \delta\Lambda^\alpha = 0, \tag{11}$$

which resembles the fermionic symmetry of twistor–like component actions for super–p–branes [1, 18] (the braces {...} denote symmetrization of the indices).

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The algebra of the transformations (10), (11) is closed.

The equations of motion derived from (8) are

$$\Pi^{\alpha\beta}\Lambda_\beta \equiv (DX^{\alpha\beta} - 2iD\Theta^{\{\alpha}\Theta^{\beta\}})\Lambda_\beta = 0, \tag{12}$$

$$\Lambda_\beta D\Theta^\beta = 0, \tag{13}$$

$$\Lambda_{\{\alpha}D\Lambda_{\beta\}} = 0, \tag{14}$$

The general solutions to (12) and (13) are, respectively,

$$\Pi^{\alpha\beta} = \Psi(\tau,\eta)\Lambda^\alpha\Lambda^\beta, \tag{15}$$

$$D\Theta^\alpha = a(\tau,\eta)\Lambda^\alpha, \tag{16}$$

At the same time, from (14) it follows that

$$D\Lambda^\alpha = 0. \tag{17}$$

On the mass shell (15) – (17) the fermionic superfield Ψ and the bosonic superfield a transform under (11), (10) and (9) as follows:

$$\delta\Psi = Db - \frac{1}{2}\partial_\tau\Xi\Psi - 2ia\kappa, \qquad \delta a = D\kappa - \frac{1}{2}\partial_\tau\Xi a, \tag{18}$$

Hence, one can fix a gauge

$$\Psi = 0, \qquad a = 1, \tag{19}$$

at which (15) and (16) are reduced, respectively, to

$$\Pi^{\alpha\beta} = 0, \tag{20}$$

$$D\Theta^\alpha = \Lambda^\alpha. \tag{21}$$

This gauge [1] is conserved under the κ–transformations reduced to the worldline supersymmetry transformations

$$D\kappa - \frac{1}{2}\partial_\tau\Xi = D(\kappa + \frac{i}{2}D\Xi) = 0. \tag{22}$$

As a result the twistor superfield Λ^α is expressed in terms of $D\Theta^\alpha$ and does not carry independent degrees of freedom, and in the gauge (19) the equations for $X^{\alpha\beta}$ and Θ^α coincide with those in the conventional twistor–like formulation (2) [1, 7].

Thus we conclude that the doubly supersymmetric action (8) is classically equivalent to (2) and describes the massless $N = 1$ superparticle.

The relationship between the two actions can be understood using the following reasoning. It was shown in [6] that for $n = 1$ the action (2) is classically equivalent to

$$S = \int d\tau d\eta (P_{\alpha\beta}\Pi^{\alpha\beta} - \frac{1}{2}EP_{\alpha\beta}P^{\alpha\beta}) \tag{23}$$

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[1] Note, that the gauge choice $a = 0$ in Eq. (19) is inadmissible since then from (15), (16) it would follow that $\frac{d}{d\tau}X^m|_{\eta=0} = 0$, which is, in general, incompatible with boundary conditions $X^m(\tau_1)|_{\eta=0} = x_1$, $X^m(\tau_2)|_{\eta=0} = x_2$.

due to the existence of the following counterparts of the transformations (11), (10) [6]

$$\delta X^{\alpha\beta} = \tilde{b} P^{\alpha\beta}, \qquad \delta E = D\tilde{b}, \qquad \Theta^{\alpha} = 0, \tag{24}$$

$$\delta X^{\alpha\beta} = 2i\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \qquad \delta E = -2i\kappa_{\alpha}D\Theta^{\alpha}, \qquad \delta\Theta^{\alpha} = \kappa_{\beta}P^{\beta\alpha}, \tag{25}$$

which allow one to put the Grassmann superfield $E(\tau, \eta)$ equal to zero globally on the worldline superspace. [2]

At the same time the variation of (23) with respect to $E(\tau, \eta)$ leads to the equation

$$P^{\alpha\beta} P_{\alpha\beta} = 0, \tag{26}$$

which can be solved as

$$P_{\alpha\beta} = \Lambda_{\alpha}\Lambda_{\beta} \tag{27}$$

with Λ_{α} being an arbitrary bosonic spinor superfield. Substituting (27) into (23) we obtain the action (8).

Thus, we have constructed a version of the twistor–like formulation of the massless $N = 1$, $D = 3$ superparticle based on eq. (8) with all symmetries of the model being irreducible. Action (8) looks very much like a worldline superfield generalization of the supertwistor action by Ferber [23].

One can even rewrite (8) in a complete supertwistor form [24, 25] by introducing the second bosonic spinor component and the Grassmann component of the supertwistor [23]:

$$M^{\alpha} = X^{\alpha\beta}\Lambda_{\beta}, \qquad \Upsilon = \Theta^{\alpha}\Lambda_{\alpha}. \tag{28}$$

Then, with taking into account the constraints (28), the action (8) takes the form

$$S = \int d\tau d\eta (\Lambda_{\alpha} D M^{\alpha} - D\Lambda_{\alpha} M^{\alpha} - 2i\Upsilon D\Upsilon).$$

3 D=4, N=1 superparticles with n=2 SUSY on the worldline.

In contrast to the superparticle formulation based on the action (2), the straightforward generalization of the D=3 action (11) to the cases D=4,6,10 seems not possible. Indeed, the measure of integration in these latter cases is even and since the $\Pi_q^m \equiv D_q X^m - iD_q\Theta\gamma^m\Theta$ is odd P_m^q must be odd as well. Thus, the Lagrange multiplier can not be replaced by the bilinear combination of bosonic spinor superfields.

But there is another formulation of D=4 superparticle theory [1, 4, 5] in terms of worldline chiral superfields $X_{L,R}{}^{\alpha\dot\beta} = X_{L,R}^m \sigma_m^{\alpha\dot\beta}$ and Θ^{α}, $\bar\Theta^{\dot\alpha}$.

It is based on the concept of double analyticity [9, 5], which means that the coordinates of the "analytical" and "antianalytical" subspaces $(X_L{}^m, \Theta^{\alpha})$ and $(X_R{}^m, \bar\Theta^{\dot\alpha})$ of the target

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[2]Note that in contrast to (11) the transformations of eq. (25) correspond to an infinite reducible κ–symmetry [13, 14, 15].

superspace $(X^{\alpha\dot\beta} = 1/2(X_L{}^{\alpha\dot\beta} + X_R{}^{\alpha\dot\beta}), \Theta^\alpha, \bar\Theta^{\dot\alpha})$ [26] should be considered as the worldline chiral superfields (the bar denotes complex conjugation).

$$X_L{}^{\alpha\dot\beta} = X_L{}^{\alpha\dot\beta}(\tau_L = \tau + i\eta\bar\eta, \eta), \qquad \Theta^\alpha = \Theta^\alpha(\tau_L, \eta); \tag{29}$$

$$X_R{}^{\alpha\dot\beta} = X_R{}^{\alpha\dot\beta}(\tau_R = \tau - i\eta\bar\eta, \bar\eta), \qquad \bar\Theta^{\dot\alpha} = \bar\Theta^{\dot\alpha}(\tau_R, \bar\eta), \tag{30}$$

then

$$DX_R{}^{\alpha\dot\beta} = 0 = D\bar\Theta^{\dot\alpha}, \qquad \bar D X_L{}^{\alpha\dot\beta} = 0 = \bar D\Theta^\alpha, \tag{31}$$

where $D_q = (D, \bar D)$

$$D = \frac{\partial}{\partial\eta} + i\bar\eta\partial_\tau = \overline{(\bar D)}$$

are the worldline Grassmann derivatives. In the central basis of the target superspace the superparticle coordinates will satisfy the geometrodynamical condition

$$\Pi_q^{\alpha\dot\alpha} \equiv D_q X^{\alpha\dot\alpha} - i(D_q\Theta^\alpha)\bar\Theta^{\dot\alpha} - i(D_q\bar\Theta^{\dot\alpha})\Theta^\alpha = 0 \tag{32}$$

if the embedding of the worldline superspace into the target superspace is defined by the chirality condition (and vice versa)

$$Y^{\alpha\dot\beta} \equiv \frac{i}{2}(X_L{}^{\alpha\dot\beta} - X_R{}^{\alpha\dot\beta}) - \Theta^\alpha\bar\Theta^{\dot\beta} = 0. \tag{33}$$

Equation (33) can be obtained from the action with a *bosonic* Lagrange multiplier

$$S = \int d\tau\, d\eta\, d\bar\eta\, P_{\alpha\dot\beta}(\frac{i}{2}(X_L{}^{\alpha\dot\beta} - X_R{}^{\alpha\dot\beta}) - \Theta^\alpha\bar\Theta^{\dot\beta}). \tag{34}$$

Instead of (34) we consider

$$S = \int d\tau\, d\eta\, d\bar\eta\, \Lambda_\alpha\bar\Lambda_{\dot\beta}(\frac{i}{2}(X_L{}^{\alpha\dot\beta} - X_R{}^{\alpha\dot\beta}) - \Theta^\alpha\bar\Theta^{\dot\beta}), \tag{35}$$

where $\Lambda_\alpha(\tau, \eta, \bar\eta)$ and $\bar\Lambda_{\dot\alpha}(\tau, \eta, \bar\eta)$ are commuting spinor superfields.

The action (35) possesses N=1 target space supersymmetry, n=2 local worldline supersymmetry

$$\delta\eta = \frac{i}{2}\bar D\Xi(\tau_L, \eta, \bar\eta), \qquad \delta\tau_L = \Xi + \bar\eta\bar D\Xi$$

,

$$\delta D = -\frac{1}{2}(\partial_\tau\Xi)D + \frac{i}{4}[D, \bar D]\Xi D, \tag{36}$$

U(1) symmetry

$$\delta\Lambda^\alpha = i\varphi\Lambda^\alpha,$$

and also has two additional symmetries analogous to (10) and (11):

$$\delta X_L{}^{\alpha\dot\beta} = \bar D(b(\tau, \eta, \bar\eta)\Lambda^\alpha\bar\Lambda^{\dot\beta}), \qquad \delta X_R{}^{\alpha\dot\beta} = \overline{(\delta X_L{}^{\alpha\dot\beta})}, \tag{37}$$

$$\delta\Lambda^\alpha = 0 = \delta\Theta^\alpha, \qquad \delta\bar\Lambda^{\dot\alpha} = 0 = \delta\bar\Theta^{\dot\alpha},$$

and

$$\delta X_L{}^{\alpha\dot\beta} = \bar{D}(\bar\kappa(\tau,\eta,\bar\eta)\Lambda^\alpha\bar\Theta^{\dot\beta}), \qquad \delta X_R{}^{\alpha\dot\beta} = D(\kappa(\tau,\eta,\bar\eta)\Theta^\alpha\bar\Lambda^{\dot\beta}), \tag{38}$$

$$\delta\Theta^\alpha = \frac{i}{2}\bar{D}(\bar\kappa\Lambda^\alpha), \qquad \delta\bar\Theta^{\dot\alpha} = -\frac{i}{2}D(\kappa\bar\Lambda^{\dot\alpha}).$$

In contrast to the D=3 case both, $b(\tau,\eta,\bar\eta)$ and $\kappa(\tau,\eta,\bar\eta)$, are complex bosonic super-fields (compare with [6]).

To vary the action (35) with respect to the superfields $X_{L,R}^m$, Θ^α, $\bar\Theta^{\dot\alpha}$, the chirality condition (31) should be taken into account. The simplest way to do this is to solve (31) using the nilpotency of the covariant spinor derivatives ($DD = 0 = \bar{D}\bar{D}$), i.e. to make use of the representation

$$X_L = \bar{D}\bar\psi, \qquad X_R = D\psi$$

,

$$\Theta^\alpha = \bar{D}t^\alpha, \qquad \bar\Theta^{\dot\alpha} = D\bar{t}^{\dot\alpha}, \tag{39}$$

and then vary with respect to unrestricted ("prepotential") superfields $\bar\psi$, ψ, t, $\bar{t}$.

The equations of motion of X_L and X_R

$$\bar{D}(\Lambda_\alpha\bar\Lambda_{\dot\beta}) = 0 = D(\Lambda_\alpha\bar\Lambda_{\dot\beta}) \tag{40}$$

read that the real light–like vector $\Lambda_\alpha\bar\Lambda_{\dot\alpha}$ is constant [3].

Fixing the gauge with respect to the U(1) transformations, we get

$$\Lambda_\alpha = const, \qquad \bar\Lambda_{\dot\alpha} = const. \tag{41}$$

Taking into account (41), the equations of motion of Θ^α and $\bar\Theta^{\dot\alpha}$

$$\delta S/\delta t^\alpha = 0, \qquad \delta S/\delta\bar{t}^{\dot\alpha} = 0$$

are reduced to

$$\Lambda_\alpha\bar\Lambda_{\dot\beta}\bar{D}\bar\Theta^{\dot\beta} = 0, \qquad \Lambda_\alpha\bar\Lambda_{\dot\beta}D\Theta^\alpha = 0 \tag{42}$$

and, in an assumption that the components of Λ_α are not to be zero simultaneously (which is a convention of the twistor approach), result in

$$\bar\Lambda_{\dot\alpha}\bar{D}\bar\Theta^{\dot\alpha} = 0, \qquad \Lambda_\alpha D\Theta^\alpha = 0. \tag{43}$$

Much simpler is the derivation of the equations

$$\delta S/\delta\Lambda_\alpha = 0 = \delta S/\delta\bar\Lambda_{\dot\alpha},$$

which have the form

$$\bar\Lambda_{\dot\beta}Y^{\alpha\dot\beta} = 0, \qquad \Lambda_\alpha Y^{\alpha\dot\beta} = 0. \tag{44}$$

The general solution to eqs. (43) and (44) is

$$D\Theta_\beta = \bar{a}(\tau,\eta,\bar\eta)\Lambda_\beta, \qquad \bar{D}\bar\Theta_{\dot\beta} = a(\tau,\eta,\bar\eta)\bar\Lambda_{\dot\beta}, \tag{45}$$

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[3] This constant light–like vector is just the momentum of the massless superparticle

$$Y_{\alpha\dot\beta} = \xi(\tau,\eta,\bar\eta)\Lambda_\alpha\bar\Lambda_{\dot\beta}, \tag{46}$$

where $a(\bar a)$ are (anti)chiral parameters and ξ is real.

On the mass shell the bosonic superfields ξ, a transform under (36), (37) and (38) as follows:

$$\delta\xi = \frac{i}{2}(b-\bar b) + (-\frac{1}{2}\partial_\tau\Xi + \frac{i}{4}[D,\bar D]\Xi)\xi, \tag{47}$$

$$\delta a = \frac{i}{2}\bar D D\kappa + (-\frac{1}{2}\partial_\tau\Xi + \frac{1}{4}[D,\bar D]\Xi)a,$$

which allows one to fix a gauge in the following form

$$\xi = 0, \qquad a = 1. \tag{48}$$

As in the case of D=3 superparticle the gauge fixing $a = 0$ is incompatible with boundary conditions (see the footnote 2 in section 3).

In the gauge (48) the equations of motion are reduced to (33) (which is the geometro-dynamical condition that appeared in a standard doubly supersymmetric formulation of the D=4 superparticle [1, 5]), and

$$D\Theta_\beta = \Lambda_\beta \qquad \bar D\bar\Theta_{\dot\beta} = \bar\Lambda_{\dot\beta}. \tag{49}$$

Eqs. (49) identify Λ_β $(\bar\Lambda_{\dot\beta})$ with $D\Theta_\beta$ $(\bar D\bar\Theta_{\dot\beta})$.

In a completely supertwistor form the action (35) looks as follows

$$S = \int d\tau d\eta d\bar\eta(\Lambda_\alpha M^\alpha - \bar\Lambda_{\dot\alpha}\bar M^{\dot\alpha} - \Upsilon\bar\Upsilon),$$

where the supertwistor components Λ_α, M^α, Υ are constrained to be

$$M^\alpha = \frac{i}{2}X_L^{\alpha\dot\beta}\bar\Lambda_{\dot\beta}, \qquad \Upsilon = \Lambda_\alpha M^\alpha.$$

4   n=4 superfield formulation of D=6 N=1 superparticles.

As in the case of D=4, there is a description of D=6, N=1 superparticle [5], based on the double analyticity concept. An analytic superspace

$$(X_A^m = X^m + i\Theta^i\gamma^m\Theta^j w_i^+ w_j^-, \Theta_\alpha^+ = \Theta_\alpha^i w_i^+, w_i^\pm) \tag{50}$$

is chosen in the D=6, N=1 harmonic superspace

$$(X^m, \Theta_\alpha^i, w_i^\pm), \tag{51}$$

where $w_i^\pm$ are the harmonic variables which parametrize the coset $SU(2)/U(1) \simeq CP^1 \simeq S^2$ [27] (i=1,2 is the SU(2) index and α=1,...,4). Note that Θ_α^i is a so called $SU(2)$ Majorana–Weyl spinor (see [32] for the details).

Worldline superspace is also assumed to be a harmonic (d=1, n=4) superspace parametrized by

$$(\tau, \eta^i, \bar{\eta}_i, u_i^{\pm}),\tag{52}$$

and its analytic subspace being parametrized by

$$(\tau_A, \eta^+, \bar{\eta}^+, u_i^{\pm})\tag{53}$$

$$\tau_A = \tau + i\eta^{(i}\bar{\eta}^{j)}u_i^+ u_j^-\tag{54}$$

$$\eta^{\pm} = \eta^i u_i^{\pm}, \qquad \bar{\eta}^{\pm} = \bar{\eta}^i u_i^{\pm}.$$

The double analyticity principle claims that the coordinates of the analytic superspace (50) depend only on the analytic superspace coordinates (53) of the worldline.

In this respect we note that, as it was demonstrated in [5], the target space and worldline $SU(2)/U(1)$ harmonic coordinates can be identified. This means that the embedding of the worldline harmonic superspace into the target harmonic superspace is realized in such a way that the harmonic sectors $SU(2)/U(1)$ coincide.

$$X_A^m = X_A^m(\tau_A, \eta^+, \bar{\eta}^+, u_i^{\pm})$$

$$\Theta_{\alpha}^+ = \Theta_{\alpha}^+(\tau_A, \eta^+, \bar{\eta}^+, u_i^{\pm})$$

$$w_i^{\pm} = u_i^{\pm}.\tag{55}$$

Further, the embedding is specified by the geometrodynamical condition in the following form [5]

$$D^{++}X_A^m - i\Theta^+\gamma^m\Theta^+ = 0\tag{56}$$

and by the restriction on the superfield Θ_{α}^+

$$D^{++}\Theta_{\alpha}^+ = 0,\tag{57}$$

where D^{++} is a harmonic derivative in the analytic basis

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + i\eta^+\bar{\eta}^+\frac{\partial}{\partial \tau_A}.\tag{58}$$

To get (56) and (57) as consequences of superparticle dynamics the authors of [5] proposed the action

$$S = -i\int du\, d\tau_A\, d\eta^+\, d\bar{\eta}^+[P^{\gamma\delta}(D^{++}X_{A\gamma\delta} - i\Theta_{\gamma}^+\Theta_{\delta}^+)$$

$$+ P^{-\alpha}D^{++}\Theta_{\alpha}^+]\tag{59}$$

which contains two Lagrange multiplier superfields P_m and $P^{-\alpha}$.

A superfield action $a\ la$ Ferber–Shirafuji looks as follows

$$S = -i\int du\, d\tau_A\, d\eta^+\, d\bar{\eta}^+[\varepsilon^{\alpha\beta\gamma\delta}\Lambda_{\alpha}^i\Lambda_{\beta i}(D^{++}X_{A\gamma\delta} - i\Theta_{\gamma}^+\Theta_{\delta}^+)$$

$$+ P^{-\alpha}D^{++}\Theta_{\alpha}^+],\tag{60}$$

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where $\Lambda_\alpha^i = \Lambda_\alpha^i(\tau_A, \eta^+, \bar\eta^+, u_i^\pm)$ are commuting analytic spinor superfields.

The action (60) possesses N=1 global target space supersymmetry

$$\delta X_A^m = 2i\varepsilon_\alpha^-(\gamma^m)^{\alpha\beta}\Theta_\beta^+, \qquad \delta\Theta_\alpha^+ = \varepsilon_\alpha^+ \tag{61}$$

$$\delta P^{-\alpha} = -2i\varepsilon^{\alpha\beta\gamma\delta}\Lambda_\beta^i\Lambda_{i\gamma}\varepsilon_\delta^j u_j^-$$

with an odd constant parameter $\varepsilon_\alpha^i = u^{+i}\varepsilon_\alpha^- + u^{-i}\varepsilon_\alpha^+$. Eq. (60) also has n=4 local worldline supersymmetry with a local $SU(2)$ automorphism group, bosonic superfield symmetry

$$\delta X_{A\gamma\delta} = \mathcal{B}(\tau_A, \eta^+, \bar\eta^+, u)\Lambda_\gamma^i\Lambda_{\delta i}, \tag{62}$$

and another local $SU(2)$ symmetry which acts on Λ_α^i

$$\Lambda_\alpha^{'i} = U^i{}_j\Lambda_\alpha^j. \tag{63}$$

The equations of motion derived from the action (60) have the following form:

$$D^{++}\Theta_\alpha^+ = 0 \tag{64}$$

$$\varepsilon^{\alpha\beta\gamma\delta}\Lambda_{\beta i}(D^{++}X_{A\gamma\delta} - i\Theta_\gamma^+\Theta_\delta^+) = 0 \tag{65}$$

$$\varepsilon^{\alpha\beta\gamma\delta}D^{++}(\Lambda_\alpha^i\Lambda_{\beta i}) = 0 \tag{66}$$

$$i\varepsilon^{\alpha\beta\gamma\delta}\Lambda_\alpha^i\Lambda_{\beta i}\Theta_\gamma^+ + D^{++}P^{-\delta} = 0 \tag{67}$$

The solutions to the equations (64), (65) and (66) are, respectively:

$$\Theta_\alpha^+ = \theta_\alpha^i u_i^+ + \eta^+\tilde\lambda_\alpha + \bar\eta^+\tilde{\bar\lambda}_\alpha - i\eta^+\bar\eta^+\dot\theta_\alpha^i u_i^-, \tag{68}$$

$$D^{++}X_{A\gamma\delta} - i\Theta_\gamma^+\Theta_\delta^+ = \xi(\tau_A, \eta^+, \bar\eta^+, u)\Lambda_\gamma^i\Lambda_{\delta i} \tag{69}$$

and

$$\varepsilon^{\alpha\beta\gamma\delta}\Lambda_\alpha^i\Lambda_{\beta i} = \varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i} = const, \tag{70}$$

where $\lambda_\alpha^i = \Lambda_\alpha^i|_{\eta=0}$.

Using the additional symmetry (62) we can choose the gauge $\xi = 0$ in which the geometrodynamical condition (65) has the standard form

$$D^{++}X_{A\gamma\delta} - i\Theta_\gamma^+\Theta_\delta^+ = 0. \tag{71}$$

To find the solution to eq. (67) let us present $P^{-\alpha}$ in the component form

$$P^{-\alpha} = \Phi^{-\alpha} + \eta^+\zeta^{--\alpha} + \bar\eta^+\bar\zeta^{--\alpha} + i\eta^+\bar\eta^+\psi^{---\alpha}. \tag{72}$$

Substituting (72) and (66) into (67) and expending the latter in the powers of η^+, $\bar\eta^+$ we obtain the following component equations and their solutions:

$$D^{++}\psi^{---\delta} = -\varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\dot\theta_\alpha^i u_i^- \Rightarrow$$

$$\begin{cases} D^{++}\psi^{---\delta} = 0 \\ \varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\dot\theta_\alpha^i u_i^- = 0 \end{cases} \Rightarrow \quad \psi^{---\delta} = 0 \tag{73}$$

$$D^{++}\zeta^{--\delta} = -i\varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\tilde{\lambda}_\gamma \Rightarrow$$

$$\begin{cases} D^{++}\zeta^{--\delta} = 0 \Rightarrow \zeta^{--\delta} = 0 \\ \varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\tilde{\lambda}_\gamma = 0 \end{cases} \tag{74}$$

$$D^{++}\Phi^{-\delta} = -i\varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\theta_\gamma^k u_k^+ \Rightarrow$$

$$\begin{cases} \Phi^{-\delta} = -i\varepsilon^{\alpha\beta\gamma\delta}\lambda_\alpha^i\lambda_{\beta i}\theta_\gamma^k u_k^- \\ \dot{\Phi}^{-\delta} = 0 \end{cases} \tag{75}$$

The relation between the superpartner $\tilde{\lambda}_\alpha$ of the Grassmann coordinate θ_α^i and λ_α^i is established with the help of the second equation in (74)

$$\tilde{\lambda}_\alpha = a_i(\tau_A)\lambda_\alpha^i \qquad \bar{\tilde{\lambda}}_\alpha = \bar{a}_i\lambda_\alpha^i. \tag{76}$$

Using the local $SU(2)$ transformations of worldline n=4 SUSY it is possible to choose the gauge $a_1 = a$, $\bar{a}_2 = ia$ and identify $\tilde{\lambda}_\alpha$, $\bar{\tilde{\lambda}}_\alpha$ with the doublet $a\lambda_\alpha^i$ of the unbroken local $SU(2)$ (63). Then, as it is easy to see, the geometrodynamical condition (65) reproduces at the component level the solution to the mass shell condition

$$\dot{x}^m - 2i\dot{\theta}^i\gamma^m\theta_i = 2a^2\lambda^i\gamma^m\lambda_i, \tag{77}$$

and upon redefining the fields and eliminating auxiliary variables one arrives at the conventional formulation of the D=6, N=1 massless superparticle.

Herein we will not consider a complete supertwistor form of the action (60) since the definition of the supertwistor associated with the harmonic D=6, N=1 superspace requires additional studies.

5 Conclusion

We have considered the formulation of N=1 superparticle mechanics in D=3,4 and 6, which is manifestly supersymmetric on the worldline and in the target superspace. For the construction of the action we have used only geometrical objects that characterize the properties of embedding the worldline superspace into the target superspace, such as target superspace coordinates of the superparticle and twistor components. The action does not contain the Lagrange multipliers which may cause the problem of infinite reducible symmetries, and, in fact, is a worldline superfield generalization of the supertwistor description of superparticle dynamics [23, 24, 25].

The generalization of the present formulation to the case of N=1, D=10 superparticles and D=3,4,6 and 10 superstrings turned out to be not straightforward and revealed rather interesting and deep connection of the twistor–like formulation of super–p–branes with the geometrical [33] and the group–manifold [34] approach. This problems are considered elsewhere [19, 20].

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